IV. *A Letter from the Rev.* Nevil Maſkelyne, *A. M. Fellow of* Trinity *College, in the Univerſity of* Cambridge, *and F. R. S. to the Rev. Dr.* Birch, *Secretary to the Royal Society; containing a Theorem of the Aberration of the Rays of Light refracted through a Lens, on account of the Imperfection of the ſpherical Figure.*

Reverend Sir,

Read Jan. 22, 1761.

ABOUT two years ago, becoming acquainted with Mr. Dollond's curious diſcovery in optics, of correcting the aberration of the rays of light ariſing from the different refrangibility of the different ſorts of rays, by a combination of two different kinds of glaſs; and learning from him, in converſation, that he had invented a theorem, ſhewing the quantity of the aberration of the rays refracted through a lens, on account of the imperfection of the ſpherical figure; by the application of which, he was able to make the aberrations of the combined concave and convex object lenſes perfectly equal to, and conſequently to correct one another; I was deſirous of being more minutely acquainted with this farther great improvement in optics; and Mr. Dollond accordingly readily offered to gratify my curioſity. But in the mean while that he was looking over his papers, in order to lay them before me, having leiſure, I ſet about the inveſtigation of a ſimilar theorem myſelf; which having

completed,

completed, I interchanged with Mr. Dollond for his theorem, he taking mine, and I taking his. Our theorems, though ſimilar, were not exactly the ſame; but, by reduction to the ſame form, I inferred his theorem from mine; which gave me a farther confidence of the exactneſs of both.

I have here ſent you my theorem, and deſire, that you will lay it before the Royal Society.

I am, SIR,

Your moſt obedient

humble ſervant,

Nevil Maſkelyne.

Prince Henry, St. Helen's Road,
Jan. 16, 1761.

LET the form of the lens aſſumed, in the inveſtigation of the theorem, be a meniſcus, the radius of whoſe convex ſurface is greater than that of its concave ſurface; and the center of whoſe two ſurfaces lie on the ſame ſide of the lens, as the radiant point, from which the rays diverge, that fall thereon. The ray falling on the extreme part of the lens will, after refraction, diverge from a point before the lens, nearer thereto than the geometrical focus of rays diverging from the ſame radiant point, and paſſing indefinitely near the vertex.

Let Q expreſs the diſtance of the radiant point, before the lens, from its vertex; R, the radius of concavity of the ſurface, on which the rays firſt fall; and *r*, the radius of convexity of the ſecond ſurface; F, the principal focus, or the focus of parallel rays;

which

which will be on the ſame ſide of the lens, as the incident rays; becauſe R, the radius of the concave ſurface, is ſuppoſed leſs than r, the radius of the convex ſurface. Let the ratio of m to n be the ſame with that of the ſine of incidence to the ſine of refraction of rays paſſing out of air into glaſs, and let Y expreſs the ſemidiameter of the aperture of the lens; the angular aberration of the ray falling on the extremity of the lens, or the angle made between this ray, after being refracted through the extremity of the lens, and another ray or line, ſuppoſed to be drawn from the ſame extremity of the lens, to the geometrical focus of rays diverging from the ſame radiant point, and paſſing indefinitely near the vertex of the lens, expreſſed in meaſures of the arc of a circle to the radius unity, will be

$$\frac{\overline{m^3 - 2m^2n + 2n^3} \times Y^3}{\overline{m-n}^2 \times 2m \times F^3} + \frac{\overline{mn + 4n^2 - 2m^2} \times Y^3}{\overline{m-n} \times 2m \times F^2 r}$$
$$+ \frac{\overline{m+2n} \times Y^3}{2m \times F r^2} - \frac{\overline{4n^2 + 3mn - 3m^2} \times Y^3}{\overline{m-n} \times 2m \times QF^2}$$
$$- \frac{\overline{2m+2n} \times Y^3}{m \times QFr} + \frac{\overline{3m+2n} \times Y^3}{2m \times Q^2F}.$$

Where R, the radius of the firſt ſurface, is exterminated; and r, the radius of the ſecond ſurface, is retained:

Or, exterminating r, the radius of the ſecond ſurface, and retaining R, the radius of the firſt ſurface, the angular aberration is alſo expreſſed by

$$\frac{m^2 \times Y^3}{2 \times \overline{m-n}^2 \times F^3} - \frac{\overline{2m+n} \times Y^3}{2 \times \overline{m-n} \times F^2 R} + \frac{\overline{m+2n} \times Y^3}{2m \times F R^2}$$
$$+ \frac{\overline{3m+n} \times Y^3}{2 \times \overline{m-n} \times QF^2} - \frac{\overline{2m+2n} \times Y^3}{m \times QFR} + \frac{\overline{3m+2n} \times Y^3}{2m \times Q^2F}.$$

It may be proper to remark, that, as in theſe theorems, the principal focus is ſuppoſed to lie before the glaſs, as well as the radiant point, to adapt the theorem to other caſes, if the lens be of ſuch a form, as that its principal focus lies behind the glaſs, F muſt be taken negative: likewiſe, if the rays fall converging on the lens, or the point, to which they converge, lie behind the glaſs, Q muſt be taken negative: laſtly, if the firſt ſurface be convex, R muſt be taken negative; and if the ſecond ſurface be concave, *r* muſt be taken negative; and if, after all theſe circumſtances are allowed for, the value of the theorem comes out poſitive, the aberration is of ſuch a nature, as to make the focus of the extreme rays fall nearer the lens before it, than the geometrical focus, or farther from the lens behind it: but if the value of the theorem comes out negative, the aberration is of ſuch a kind, as to make the focus of the extreme rays fall farther from the lens before it, than the geometrical focus.

With reſpect to the application of this theorem to Mr. Dollond's combined object glaſſes, it is evident, that if the aberrations of the convex and concave lenſes added together (paying due regard to the ſigns of the theorem), are made equal to nothing, the two lenſes will perfectly correct one another: but as there are two unknown quantities unlimited in the equation, namely, the radius of one ſurface of each glaſs (for F and Q are given, as well as *m* and *n*), there is room for an arbitrary aſſumption of one of them, at the diſcretion of the theoriſt, or artiſt; which being done, there will remain a quadratic equation,

whence

whence there will reſult two values of the radius, which remains unknown, either of which will produce an aberration equal to that of the other lens.

V. *Extract of a Letter from the Abbé* De la Caille, *of the Royal Academy of Sciences at* Paris, *and F. R. S. to* William Watſon, *M. D. F. R. S. recommending to the Rev. Mr.* Nevil Maſkelyne, *F. R. S. to make at* St. Helena *a Series of Obſervations for diſcovering the Parallax of the Moon.*

Lincoln's-Inn-Fields, 8 Jan. 1761.

Read Jan. 8, 1761.

DR. Watſon lately received a letter from the Abbé De la Caille at Paris, in which he takes notice, " That although the parallax of the moon ſeems ſufficiently well deter-
" mined, by the obſervations made in 1751, in
" Europe and at the Cape of Good Hope; never-
" theleſs, an element of this importance cannot be
" too well aſcertained. He is of opinion, that Mr.
" Maſkelyne's continuance in St. Helena may be
" advantageouſly employed in making new obſerva-
" tions; ſince the baſe, upon which theſe parallaxes
" ſhould be calculated, ſhould exceed the earth's
" radius.

" That if the Royal Society does approve of his
" propoſition, and recommend to Mr. Maſkelyne
" the execution of the ſcheme of correſpondence,